



B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

PRE BOARD-1 EXAMINATION- 2025-26 MATHEMATICS (Marking Key)

Class: XII A
Date: 10 /11/25
Admission no:

Time: 3 hr
Max Marks: 80
Roll no:

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

SECTION A

The section comprises of 20 multiple choice questions (MCQs) of 1 mark each

1. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find A^{-1} .
A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. If Vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$
A) $\vec{a} \parallel \vec{b}$ B) $\vec{a} \perp \vec{b}$ C) $|\hat{a}| > |\hat{b}|$ D) $|\hat{a}| = |\hat{b}|$
3. $\int_{-1}^1 \frac{|x|}{x} dx$, $x=0$ is equal to :
A) -1 B) 0 C) 1 D) 2
4. Which of the following is not a homogeneous function of x and y .
A) $y^2 - xy$ B) $x - 3y$ C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ D) $\tan x - \sec y$
5. If $f(x) = |x| + |x - 1|$, then which of the following is correct?
A) $F(x)$ is both continuous and differentiable at $x=0$ and $x=1$.
B) $F(x)$ is differentiable but not continuous, at $x=0$ and $x=1$.
C) **$F(x)$ is continuous but not differentiable at $x=0$ and $x=1$.**
D) $F(x)$ is neither continuous neither differentiable at $x=0$ and $x=1$.
6. If A is a square matrix of order 2 such that $\det(A) = 4$. Then $\det(4\text{adj}A)$ is equal to :

- A) 16 **B) 64** C) 256 D) 512
7. If E and F are two independent events such that $P(E) = 2/3$, $P(F) = 3/7$, then $P(E/\bar{F})$ is equal to:
 A) $1/6$ B) $1/2$ **C) $2/3$** D) $7/9$
8. The absolute maximum value of the function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:
 A) 0 B) 2 **C) 4** D) 5
9. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9 \ 8 \ 7]$, which of the following is defined.
 A) **Only AB** B) Only AC C) Only BA D) All AB, BC, BA
10. If $\int \frac{2^{1/x}}{x^2} dx = k \cdot 2^{1/x} + C$, then k is equal to:
 A) **$-1/\log 2$** B) $-\log 2$ C) -1 D) $1/2$
11. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is
 A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ **C) $\frac{\pi}{3}$** D) $\frac{\pi}{2}$
12. The integrating factor of differential equation $(x+2y^2)\frac{dy}{dx} = 2y$ is :
 A) e^{y^2} **B) $\frac{1}{\sqrt{y}}$** C) $\frac{1}{y^2}$ D) $e^{y/2}$
13. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x .
 A) 0 **B) 1** C) 7 D) ± 7
14. The corner points of the feasible region in graphical representation of a LPP are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then
 A) Z is maximum at (2, 72), minimum at (15, 20)
 B) Z is maximum at (15, 20), minimum at (40, 50)
C) Z is maximum (40, 50), minimum at (15, 20)
 D) Z is maximum at (40, 50), minimum at (2, 72).
15. If A and B are invertible Matrices, then which of the following is not correct:
 A) **$(A+B)^{-1} = B^{-1} + A^{-1}$** B) $(AB)^{-1} = B^{-1}A^{-1}$ C) $\text{adj}(A) = |A|A^{-1}$ D) $|A|^{-1} = |A^{-1}|$
16. If the feasible region of a linear programming problem with objective function $Z = ax + b$, is bounded, then which of the following is the correct?
 A) It will only have maximum value
 B) it will only have minimum value
C) It will have both maximum and minimum value
 D) It will have neither maximum nor minimum value.
17. The area bounded by the curve $y^2 = x$, $x = 4$ and x- axis in first quadrant is
 A) $\int_0^4 x dx$ B) $\int_0^2 y^2 dy$ C) $2 \int_0^4 \sqrt{x} dx$ **D) $\int_0^4 \sqrt{x} dx$**
18. Find the principal value of $\sin^{-1}(-\frac{\sqrt{3}}{2})$.
 A) $\frac{\pi}{3}$ **B) $\frac{2\pi}{3}$** C) $\frac{5\pi}{3}$ D) π

Assertion and Reasoning questions: In the following two questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true and R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

- 19 Assertion (A): Let Z be the set of integers, A function $f: Z \rightarrow Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is bijective (D)

Reason (R): A function is bijective if it is both surjective and injective.

- 20 Assertion (A): $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x < 5 \end{cases}$ is continuous at $x=5$ for $k=5/2$. (D)

Reason (R): For a function f to be continuous at $x=n$. LHL = RHL = $f(n)$.

SECTION B

This section comprises of very short answer (VSA) type questions of 2 mark each.

- 21 Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$

Sol: $du/dx = 2\cos^2 x \sin 2x \log 2$, $dv/dx = \sin 2x$

$$du/dv = 2\cos^2 x \log 2$$

OR

If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

Sol: $x^2 + y^2 = \tan(a^2)$,

Diff. wrt x $2x + 2y \frac{dy}{dx} = 0$, $\frac{dy}{dx} = -x/y$

- 22 Evaluate: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

Sol: $\tan^{-1}\left[2\sin\left(2\frac{\pi}{6}\right)\right]$

$$\tan^{-1}\left[2\sin\frac{\pi}{3}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

23. The diagonals of parallelogram given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ find the area of the parallelogram.

Sol: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = 2\hat{i} + 3\hat{j} + 7\hat{k} = |\vec{a} \times \vec{b}| = \sqrt{62}$

Area of parallelogram = $\frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} \sqrt{62}$

- 24 Find the intervals in which function $f(x) = 5x^{3/2} - 3x^{5/2}$ is (i) increasing, (ii) decreasing.

Sol: $f'(x) = 15/2 x^{1/2} - 15/2 x^{3/2} = 15/2 x^{1/2} (1-x)$

Critical point is $x=0$, $x=1$.

$f(x)$ is increasing on $[0,1]$ and $f(x)$ is decreasing on $(1,\infty)$.

- 25 two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kites strings. Assuming there is no slack in the string.

Sol: Angle between the vectors a and b is given by $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{12}{\sqrt{14} \times 2\sqrt{6}} = \frac{3}{\sqrt{21}}$.

OR

Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2,1,3) and B(8,-1,0) respectively.

Sol: $\overrightarrow{AB} = 6\hat{i} - 2\hat{j} - 3\hat{k}$, Direction of vector AB is $\frac{6\hat{i}-2\hat{j}-3\hat{k}}{\sqrt{36+4+9}} = \frac{6\hat{i}-2\hat{j}-3\hat{k}}{7}$

Vector of magnitude 21 is $21 \times \frac{6\hat{i}-2\hat{j}-3\hat{k}}{7} = 3(6\hat{i} - 2\hat{j} - 3\hat{k}) = 18\hat{i} - 6\hat{j} - 9\hat{k}$

SECTION C

This section comprises of 6 short answers (SA) of mark each

- 26 The sides of an equilateral triangle is increasing at the rate of 3cm/s. At what rate its area increasing when the side of the triangle is 15 cm?

Sol: $A = \frac{\sqrt{3}}{4} s^2$ Differentiating b/s wrt t we have $\frac{dA}{dt} = \frac{\sqrt{3}}{4} 2s \frac{ds}{dt} = \frac{\sqrt{3}}{2} s \frac{ds}{dt} = \frac{\sqrt{3}}{2} 15 \times 3$
 $= \frac{45\sqrt{3}}{2}$.

- 27 Solve the following linear programming problem graphically:

Maximise $Z = x + 2y$

Subject to constraint: $x - y \geq 0$, $x - 2y \geq -2$, $x \geq 0$, $y \geq 0$.

Sol: Proper Graph, Corner points and feasible region, hence the Max Z occurs at (2, 2) with max value =6

28 $\int \frac{x + \sin x}{1 + \cos x} dx$

Sol: $\int \frac{x + \sin x}{1 + \cos x} dx = \int \frac{x}{2\cos^2 x/2} dx + \int \frac{\sin x}{2\cos^2 x/2} dx = 1/2 \int 2\sec^2 x/2 dx + \int \tan x/2 dx$

Using integration by parts we have $x \tan x/2 + c$

OR

$$\int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2\sin 2x}}$$

$$\begin{aligned}
\text{Let } I &= \int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}} \\
&= \int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin x \cdot \cos x}} \\
&= \frac{1}{2} \int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{\frac{\sin x}{\cos x} \cdot \cos^2 x}} \\
&= \frac{1}{2} \int_0^{\pi/4} \frac{dx}{\cos^4 x \sqrt{\tan x}} \\
&= \frac{1}{2} \int_0^{\pi/4} \frac{\sec^4 x \, dx}{\sqrt{\tan x}} \\
&= \frac{1}{2} \int_0^{\pi/4} \frac{\sec^2 x \cdot \sec^2 x \, dx}{\sqrt{\tan x}}
\end{aligned}$$

$$\text{Let } \tan x = t, \quad x = 0$$

$$\Rightarrow t = 0 \quad \text{and } x = \frac{\pi}{4}$$

$$\Rightarrow t = 1$$

$$\sec^2 x \, dx = dt$$

$$\begin{aligned}
\therefore I &= \frac{1}{2} \int_0^1 \frac{(1+t^2) dt}{\sqrt{t}} \\
&= \frac{1}{2} \int_0^1 (t^{-1/2} + t^{3/2}) dt \\
&= \frac{1}{2} \int_0^1 (t^{-1/2} + t^{3/2}) dt \\
&= \frac{1}{2} \left[\frac{t^{-1/2+1}}{-1/2+1} \right]_0^1 + \frac{1}{2} \left[\frac{t^{3/2+1}}{3/2+1} \right]_0^1 \\
&= \frac{1}{2} \times \frac{2}{1} [\sqrt{t}]_0^1 + \frac{1}{2} \times \frac{2}{5} [t^{5/2}]_0^1 \\
&= 1 + \frac{1}{5} = \frac{6}{5}
\end{aligned}$$

- 29 Verify that lines $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence find the shortest distance between the lines.

$$\text{Sol: } \vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ Here in both the lines vector b are not in proportion hence, Lines are not parallel,

$$d = \left| \frac{(-2\hat{i} + 4\hat{j} + 3\hat{k}) \cdot (-\hat{j} + 4\hat{k})}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}}$$

OR

During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining The bowler and the leg slip fielder.

Sol:

Identify the coordinates: B = (2, 8), W = (6, 12), F = (12, 18).

Set up the equations using the section formula: x-coordinate: $6 = (k * 12 + 2) / (k + 1)$ and y-coordinate: $12 = (k * 18 + 8) / (k + 1)$.

From the x-coordinate equation, multiply both sides by (k + 1): $6(k + 1) = k * 12 + 2$.

Simplifying gives: $6k + 6 = 12k + 2$, leading to $6k = 4$, thus $k = 2/3$.

From the y-coordinate equation, multiply both sides by (k + 1): $12(k + 1) = k * 18 + 8$.

Simplifying gives: $12k + 12 = 18k + 8$, leading to $6k = 4$, thus $k = 2/3$.

The wicketkeeper divides the line segment in the ratio 2:3.

30. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there are one gold and one silver coin. A person chooses a box random and takes out a coin, If the coin is of gold, what is the probability that the other coin in the box is also of gold?

Sol: $P(B1) = P(B2) = P(B3) = 1/3$, G; Gold coin, $P(G/B1) = 1$, $P(G/B2) = 0$, $P(G/B3) = 1/2$.

By using Bayes theorem: $P(B1/G) = 2/3$.

OR

For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was found that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in the written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Sol: $P(F) = 2/3$, $P(M) = 1/3$, D : Getting Distinction, $P(D/M) = 0.4$, $P(D/F) = 0.35$

$$P(D) = \frac{1.1}{3} = 0.3667$$

31. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x- axis, between $x = -6$ and $x = 0$, using integration.

Sol: Proper Graph and limit: -6 to -3 and -3 to 0, $A = 9/2$

SECTION D

This section comprises of 4 long answer (LA) type questions of 5 mark each.

32. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then Prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

$$\text{Put } x = \sin \theta, y = \sin \phi$$

$$\therefore \theta = \sin^{-1} x, \phi = \sin^{-1} y$$

$$\sqrt{1-\sin^2 \theta} + \sqrt{1-\sin^2 \phi} = a(\sin \theta - \sin \phi)$$

$$\cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$$

$$2 \cos \left(\frac{\theta+\phi}{2} \right) \cdot \cos \left(\frac{\theta-\phi}{2} \right) = 2a \cos \left(\frac{\theta+\phi}{2} \right) \cdot \sin \left(\frac{\theta-\phi}{2} \right)$$

$$\frac{\cos \left(\frac{\theta-\phi}{2} \right)}{\sin \left(\frac{\theta-\phi}{2} \right)} = a$$

$$\cot \left(\frac{\theta-\phi}{2} \right) = a$$

$$\frac{\theta-\phi}{2} = \cot^{-1} a$$

$$\frac{\theta-\phi}{2} = \cot^{-1} a$$

$$\theta - \phi = 2 \cot^{-1} a$$

$$\sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$$

Differentiating w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

OR

If $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ and $y = \sin \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

$$\frac{dx}{d\theta} = a \left(-\sin \theta + \frac{\sec^2 \frac{\theta}{2}}{2 \tan \frac{\theta}{2}} \right) \frac{dy}{d\theta} = a \cos \theta$$

$$= a \left(-\sin \theta + \frac{1}{\sin \theta} \right)$$

$$= a \left(\frac{-\sin^2 \theta + 1}{\sin \theta} \right)$$

$$= \frac{a \cos^2 \theta}{\sin \theta}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\frac{dy}{dx} = a \cos \theta \times \frac{\sin \theta}{a \cos^2 \theta}$$

$$\frac{dy}{dx} = \tan \theta$$

Differentiating with respect to x

$$\frac{d^2y}{dx^2} = \sec^2 \theta \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = (\sqrt{2})^2 \times \frac{\sin \theta}{a \cos^2 \theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 \times \frac{\frac{1}{\sqrt{2}}}{a \left(\frac{1}{\sqrt{2}} \right)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2\sqrt{2}}{a}$$

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 15]$.

Given,

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$f'(x) = 6x^2 - 30x + 36$$

$$f'(x) = 6(x^2 - 5x + 6)$$

$$= 6(x - 2)(x - 3)$$

Note that

$$f'(x) = 0$$

Gives,

$$x = 2 \text{ and } x = 3$$

We shall now evaluate the value of f at these points and at the end points of the interval $[1, 5]$,

At $x = 1$,

$$f(1) = 2(1^3) - 15(1)^2 + 36(1) + 1 = 24$$

At $x = 2$,

$$f(2) = 2(2^3) - 15(2)^2 + 36(2) + 1 = 29$$

At $x = 3$,

$$f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 1 = 28$$

At $x = 5$,

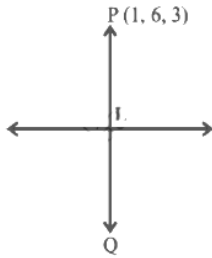
$$f(5) = 2(5)^3 - 15(5)^2 + 36(5) + 1 = 56$$

Thus,

We conclude that the absolute maximum value of f on $[1, 5]$ is 56, occurring at $x = 5$, and absolute value of f on $[1, 5]$ is 24 which occurs at $x = 1$.

34. Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A' .

Let $P(1, 6, 3)$ be the given point and let L be the foot of perpendicular from P to the given line.



The coordinates of a general point on the given line are

$$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

i.e., $x = \lambda, y = 2\lambda + 1, z = 3\lambda + 2$.

If the coordinates of L are $(\lambda, 2\lambda + 1, 3\lambda + 2)$, then the direction ratios of PL are $\lambda - 1, 2\lambda - 5, 3\lambda - 1$.

But the direction ratios of given line which is perpendicular to PL are $1, 2, 3$.

Therefore, $(\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0$, which gives $\lambda = 1$.

Hence coordinates of L are $(1, 3, 5)$.

Let $Q(x_1, y_1, z_1)$ be the image of $P(1, 6, 3)$ in the given line.

Then L is the mid-point of PQ .

Therefore,

$$\frac{x_1+1}{2} = 1$$

$$\frac{y_1+6}{2} = 3$$

$$\frac{z_1+3}{2} = 5$$

$$\Rightarrow x_1 = 1, y_1 = 0, z_1 = 7$$

Hence, the image of $(1, 6, 3)$ in the given line is $(1, 0, 7)$.

OR

Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q .

Any point P on the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$$

is $P(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$

$\therefore$ it is given that the distance between point P and Q is 7 units.

$$\Rightarrow |PQ| = 7$$

$$= |PQ|^2 = 49$$

$$\text{Now } \vec{PQ} = (\lambda - 7)\hat{i} + (4\lambda - 7)\hat{j} + (7 - 9\lambda)\hat{k}$$

$$\Rightarrow (\lambda - 7)^2 + (4\lambda - 7)^2 + (7 - 9\lambda)^2 = 49$$

$$\Rightarrow \lambda = 1$$

$\therefore$ point P will be $(-4, 1, -3)$

The equation of line joining $(-4, 1, -3)$ and $(2, 4, -1)$ will have direction ratio

$\langle 6, 3, 2 \rangle$

$\therefore$ Line joining P & Q is

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$

35. A school to allocate students into three clubs: Sports, Music and Drama, under following conditions: The number of students in sports club should be equal to the sum of the number of students in Music and Drama club. The number of students in music club should be 20 more than the number of students in sports club. The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.

Let x be the number of students in sports club.

Let y be the number of students in the music club.

Let z be the number of students in the drama club.

Given Conditions:

1. The numbers of students in the sports club is equal to the sum of the number of students in the music and drama club.

$$x = y + z$$

2. The number of students in the Music club is 20 more than half of the number of students in sports class

$$\Rightarrow y = \frac{x}{2} + 20$$

3. Total students is 180

$$\Rightarrow x + y + z = 180$$

Now we given three equation

$$x - y - z = 0$$

$$2y - x = 0$$

$$x + y + z = 180$$

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \\ 180 \end{bmatrix}$$

$$A \quad \quad \quad X = B$$

$$\Rightarrow X = A^{-1} B$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$= \frac{\begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \\ -3 & -2 & 1 \end{pmatrix}}{4}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ -\frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 0 \\ 40 \\ 180 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 90 \\ 65 \\ 25 \end{pmatrix}$$

⇒ Sports club = 90

Music club = 65

Drama club = 2533.

SECTION E

This section comprises of case study based questions of 4 mark each.

36. A technical company is designing a rectangular solar panel installation on a roof using 300m of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and the parallel to the partition be y meters.



Based on the above information, answer the following questions:

- Write the equation for the total boundary material used in the boundary and parallel to the portion in terms of x and y .
- Write the area of the solar panel as function of x .
- Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

Sol: i) $2x+3y=300$

ii) $x(100-\frac{2}{3}x)$

iii) $x=75$, $y=50$ and Area = 3750

37. A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.



Based on the above information, answer the following questions.

- i) What is the probability that a customer after availing the loan will default on the loan repayment?
- ii) A customer availing the loan, defaults on loan repayment. What is the probability that he availed the loan at variable rate of interest?

Sol: i) By Bayes Theorem: $180/1000 = 0.018$.

ii) By Bayes Theorem: $7/18$

38. A classroom teacher is keen to assess the learning of her students the concept of “Relation” taught to them. She writes the following five relations each defined on the set $A = \{1,2,3\}$: $R_1: \{(2,3), (3,2)\}$, $R_2: \{(1,2), (1,3), (3,2)\}$, $R_3: \{(1,2), (2,1), (1,1)\}$, $R_4: \{(1,1), (1,2), (3,3), (2,2)\}$, $R_5: \{(1,1), (1,2), (3,3), (2,2), (2,1), (2,3), (3,2)\}$.

The students asked to answer the following questions about the above relations:

- i) Identify the relation which is reflexive, transitive but not symmetric.
- ii) Identify the relation which is reflexive and symmetric but not transitive.
- iii) Identify the relations which are symmetric but neither reflexive nor transitive.

Sol: i) R_4

ii) No Relation

iii) R_1

****All the Best ****